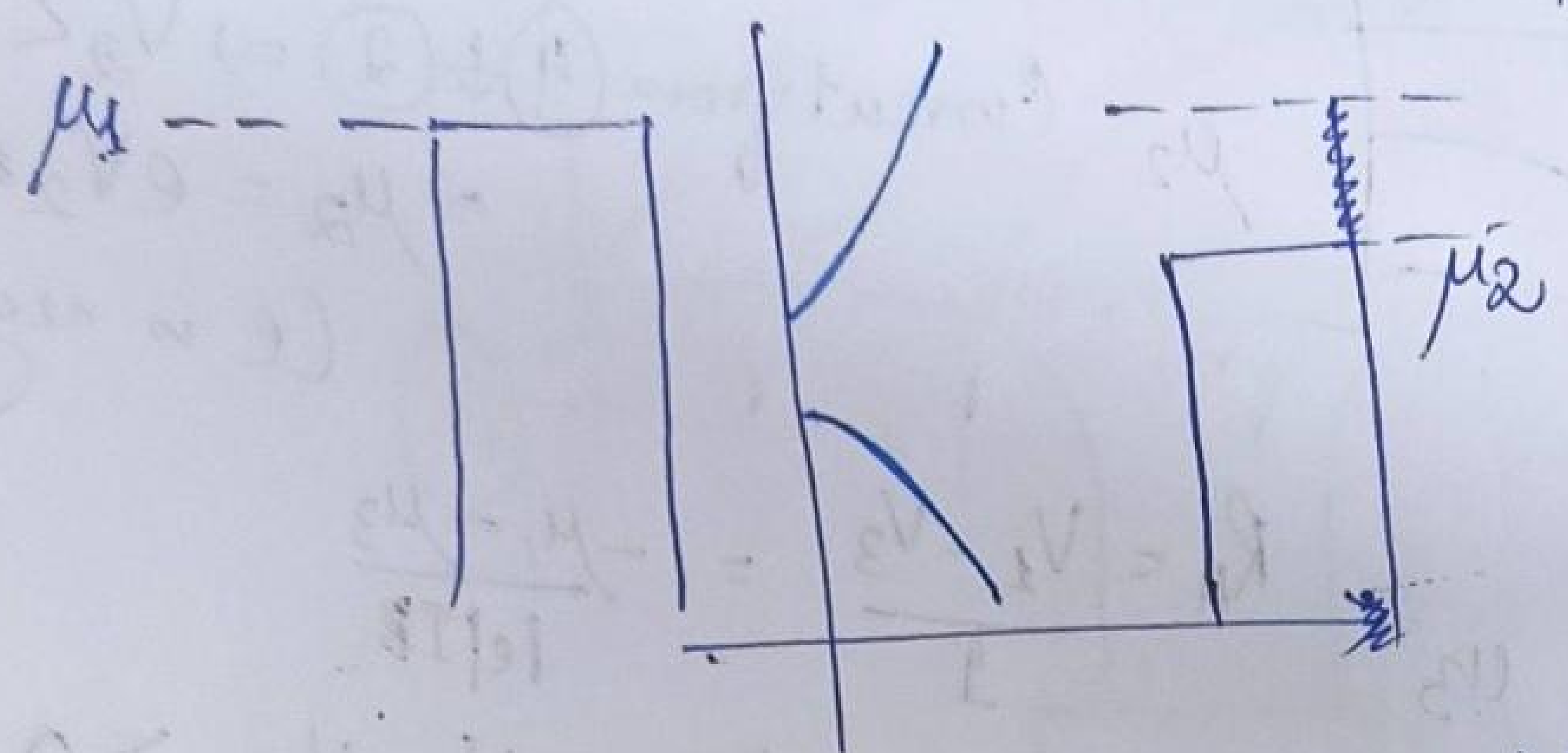
LANDAUER'S FORMULA

Electrons as wave $\Rightarrow$ conductance of the system is determined by the wave transmission properties of electrons.



$$\mu_1 - \mu_2 = qV = eV.$$

Net current $\Rightarrow$ Difference in current from left reservoir to right and the current from right to left.

$$I = I_{1\to 2} - I_{2\to 1}$$

$$= \underbrace{2}_{\text{spin degeneracy}} \frac{e}{L}\left[\sum_{k>0} v(E) f_1(E) - \sum_{k>0} v(E) f_2(E)\right]$$

Changing summation to integration

$$I = 2 \cdot \frac{e}{L} \cdot \frac{L}{2\pi}\left[\int_0^\infty v(E) f_1(E)\, dk - \int_0^\infty v(E) f_2(E)\, dk\right].$$

$$\text{or, } I = 2 \cdot \frac{e}{L} \cdot \frac{L}{2\pi}\left[\int_0^{\mu_1} \frac{1}{\hbar}\frac{dE}{dk} f_1(E)\, dk - \int_0^{\mu_2} \frac{1}{\hbar}\frac{dE}{dk} f_2(E)\, dk\right]$$

$$= 2\frac{e}{h}\int_{\mu_2}^{\mu_1}\left[f_1(E) - f_2(E)\right] dE$$

Assuming conductance at the fermi energy and $T \to 0$; we get

$$I = 2\frac{e}{h}(\mu_1 - \mu_2) = \frac{2e}{h} \times eV = \frac{2e^2 V}{h}$$

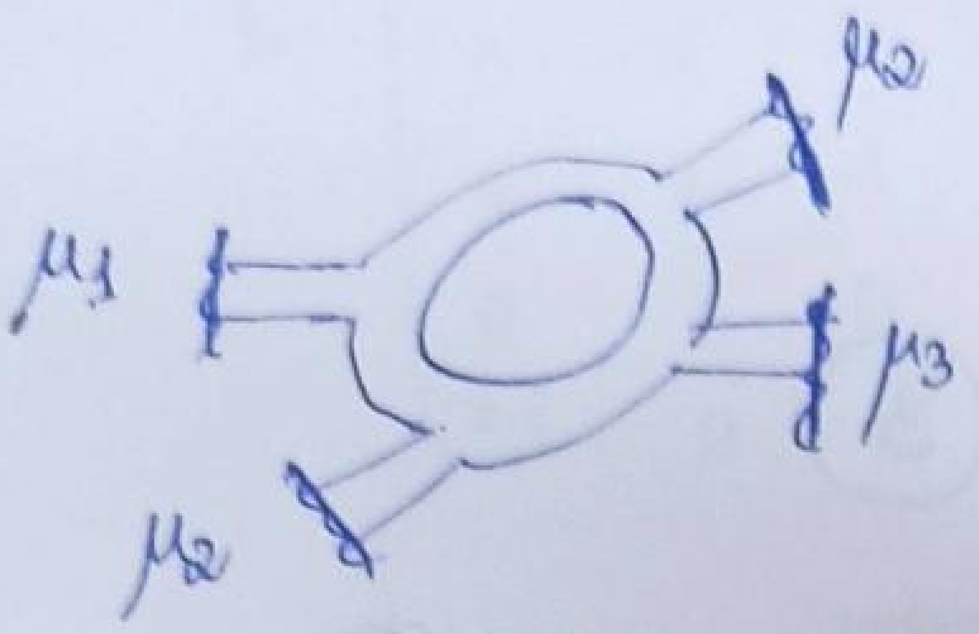
$$\text{or, } \boxed{G = \frac{I}{V} = \frac{2e^2}{h}}$$

This is for a single mode. For M modes

$$\boxed{G = \frac{2e^2}{h} \times M.} \Rightarrow \text{Landauer's formula.}$$

LANDAUER-BÜTTIKER FORMALISM

Ref: Büttiker, PRL 57, 1761 (1986)



Conductance of a multiterminal sample of arbitrary shape.

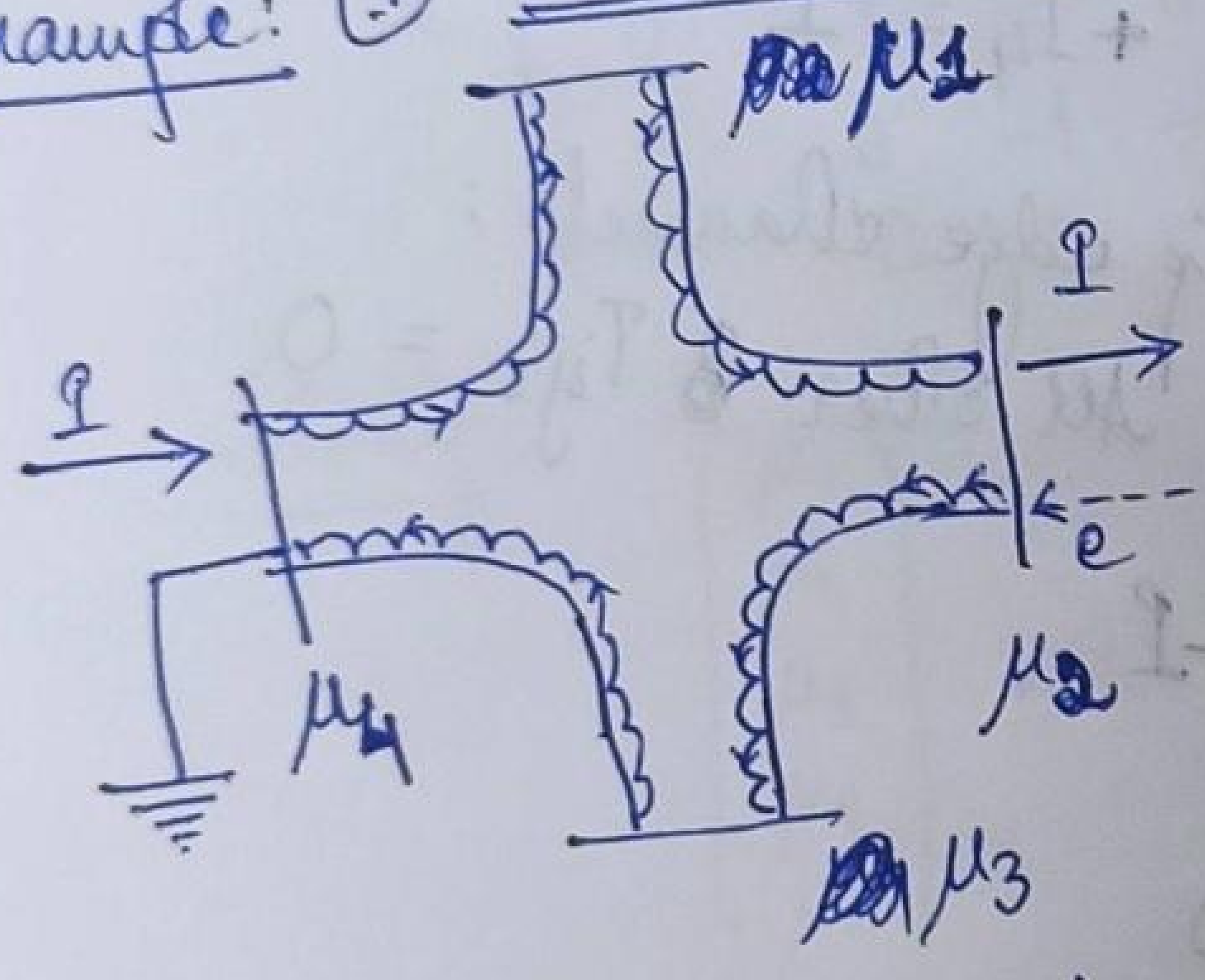
Consider contact "i" ; at chemical potential μ_i.

Current in contact "i" is given by

$$I_i = \frac{e}{h} \sum_{j \neq i} \left(T_{ji}\mu_i - T_{ij}\mu_j \right)$$

$T_{ij} \Rightarrow$ Transmission probability for the electron to go from contacts $i \to j$.

Example: (1) Hall Cross



$T_{23} = T_{34} = T_{41} = T_{12} = 1$

All other T_{ij}s $= 0$.

$$I_1 = \frac{e}{h}(\mu_1 - \mu_2) = 0 \Rightarrow \mu_1 = \mu_2$$

$$\cancel{I_2 = \frac{e}{h}(\mu_2 - \mu_3) = I} \quad I_2 = \frac{e}{h}(\mu_2 - \mu_3) = -I$$

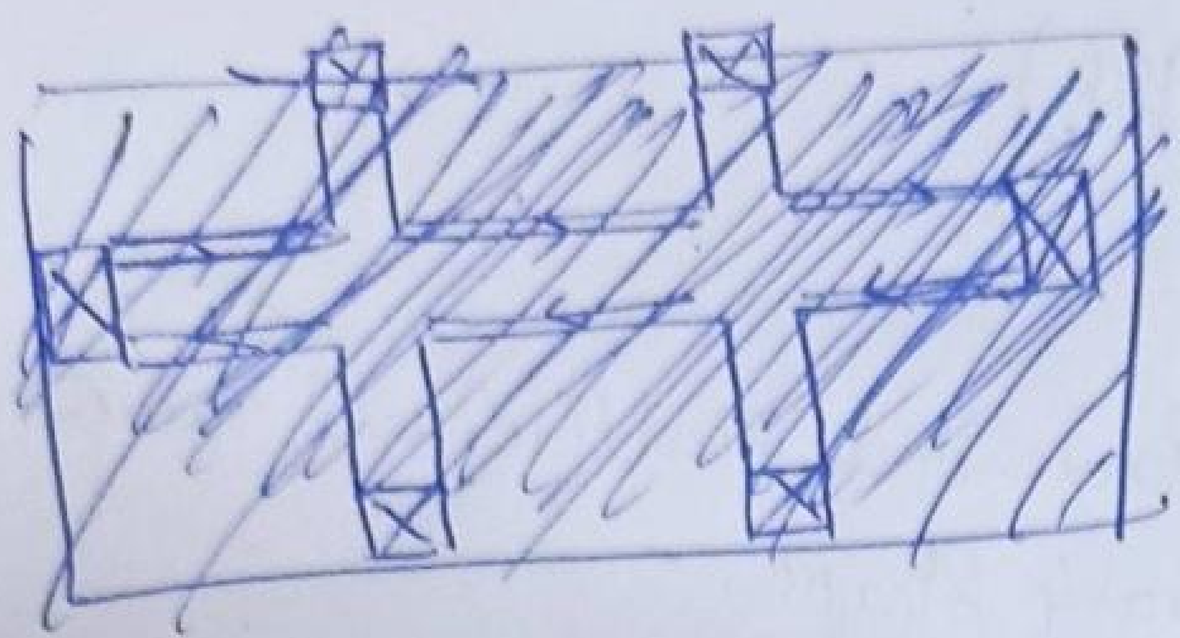
$$I_3 = \frac{e}{h}(\mu_3 - \mu_4) = 0 \Rightarrow \mu_3 = \mu_4$$

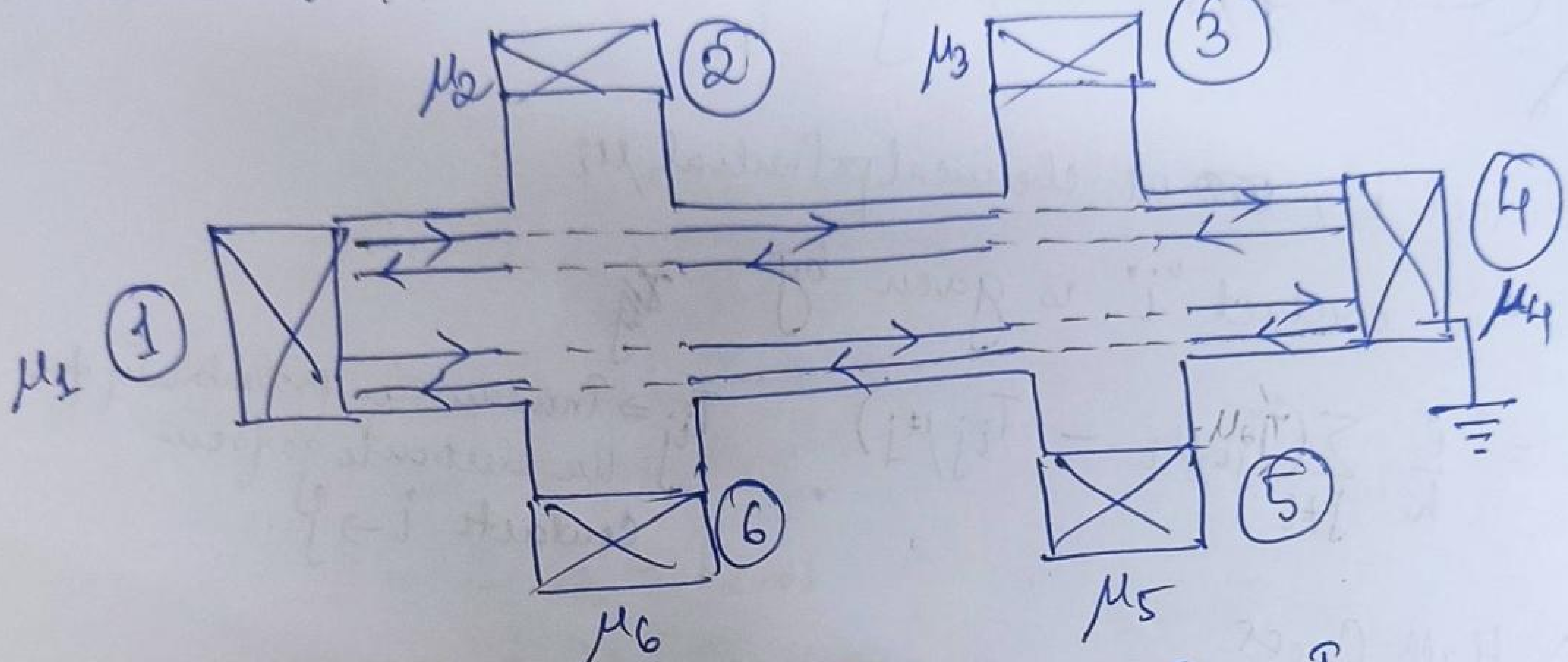
$$I_4 = \frac{e}{h}(\mu_4 - \mu_1) = I$$

$$I = \frac{e}{h}(\mu_3 - \mu_2) = \frac{e}{h} \times (-e V_{32})$$

or, $R_{Hall} = R_{23} = \dfrac{h}{e^2}$.

Example 2: Quantum Spin Hall edge Channels.





Current from contact 1 to 4 $\quad -I_1 = +I_4 = I$.

Note:- Since we have counter propagating edge channels;

$$T_{i,i+1} = T_{i+1,i} = 1; \quad \text{All other } T_{ij}\text{'s} = 0.$$

$$I_1 = \frac{e}{h}\left(2\mu_1 - \mu_2 - \mu_6\right) = -I$$

$$I_2 = \frac{e}{h}\left(2\mu_2 - \mu_1 - \mu_3\right) = 0$$

$$I_3 = \frac{e}{h}\left(2\mu_3 - \mu_2 - \mu_4\right) = 0$$

$$I_4 = \frac{e}{h}\left(2\mu_4 - \mu_3 - \mu_5\right) = +I$$

$$I_5 = \frac{e}{h}\left(2\mu_5 - \mu_4 - \mu_6\right) = 0$$

$$I_6 = \frac{e}{h}\left(2\mu_6 - \mu_5 - \mu_1\right) = 0$$

Can be writtens in matrixform

$$\frac{e}{h}\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{pmatrix} = I\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$ $\det(A)=0$; ~~Need to f~~ We need to choose on μ's as reference.
Since contact (4) is grounded; we choose $\mu_4 = 0$.
Substituting $\mu_4 = 0$ in above eq^n and performing row reduction, we get

$$\frac{e}{h}\begin{pmatrix} +2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_5 \\ \mu_6 \end{pmatrix} = I\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

Solving:-

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{pmatrix} = I\,\frac{h}{e}\begin{pmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= I\,\frac{h}{e}\begin{pmatrix} -3/2 \\ -1 \\ -1/2 \\ -1/2 \\ -1 \end{pmatrix}$$

Calculate: $R_{14,14} \Rightarrow V_{14} = (\mu_1 - \mu_4)/-e = \left(-\frac{3}{2}\frac{h}{e} I - 0\right)\cdot\frac{1}{-e}$

$$= \frac{3}{2} h/e^2 \cdot I.$$

$$R_{14,14} = \frac{V_{14}}{I} = \frac{3}{2}\frac{h}{e^2}.$$

Four terminal resistance: $R_{14,23} \Rightarrow V_{23} = (\mu_2 - \mu_3)/-e = \left(\frac{h}{e^2} - \frac{1}{2}\frac{h}{e^2}\right) I = \frac{h}{2e^2}.I$

$$\boxed{R_{14,23} = \frac{V_{23}}{I} = \frac{h}{2e^2}}$$

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